1. An airplane is flying at an altitude of 5 miles and passes directly over a radar antenna (see figure). When the plane is 10 miles away (s = 10 mi), the radar detects that the distance s is changing at a rate of 240 miles per hour. What is the speed of the plane?

\[ S = 10 \implies x^2 + 25 = 100 \]
\[ x^2 = 75 \]
\[ x = 5\sqrt{3} \]

\[ \frac{ds}{dt} = 240 \text{ mi/hr} \]

\[ \frac{dx}{dt} = \frac{s}{x} \frac{ds}{dt} = \frac{240S}{x} \]

\[ S = 10 \text{ mi} \implies \frac{dx}{dt} = \frac{240(10)}{5\sqrt{3}} \]
\[ \frac{dx}{dt} = \frac{480}{5\sqrt{3}} \text{ mi/hr} \]
2. A baseball diamond has the shape of a square with sides 90 feet long (see figure). A player running from second base to third base at a speed of 28 feet per second is 30 feet from third base. At what rate is the player’s distance \( s \) from home plate changing?

\[
s^2 = 90^2 + (90-x)^2
\]

\[
2s \frac{ds}{dt} = -2(90-x) \frac{dx}{dt}
\]

\[
\frac{ds}{dt} = \frac{x-90}{s} \frac{ds}{dt}
\]

\[
\frac{ds}{dt} = \frac{28(x-90)}{s}
\]

When \( 90-x = 30 \)

\[
\begin{align*}
\frac{ds}{dt} &= \frac{28(30)}{30 \sqrt{10}} \\
\frac{ds}{dt} &= \frac{28}{\sqrt{10}} \text{ ft/sec}
\end{align*}
\]
3. A balloon rises at a rate of 30 meters per second from a point on the ground 30 meters from an observer. Find the rate of change of the angle of elevation of the balloon from the observer when the balloon is 30 meters above the ground.

\[
\tan \theta = \frac{y}{30} \\
\sec^2 \theta \frac{d\theta}{dt} = \frac{1}{30} \frac{dy}{dt} \\
\frac{d\theta}{dt} = \cos^2 \theta
\]

\[
\frac{d\theta}{dt} = \left(\frac{12}{2}\right)^2 = \frac{1}{2} \quad \frac{d\theta}{dt} = \frac{1}{2} \text{ rad/sec}
\]

\[
y, \quad \frac{dy}{dt} = 30 \text{ m/sec}
\]

When \(y = 30\) \(\Rightarrow \theta = \frac{\pi}{4}\)
4. Water is flowing into a cone at a rate of 2 cm$^3$/min. The cone has a height of 16 cm and a radius of 4 cm. How fast is the water level rising when it is 5 cm deep and 10 cm deep?

\[
\frac{dV}{dt} = 2 \text{ cm}^3/\text{min}
\]

\[
V = \pi r^2 h
\]

\[
V = \pi (\frac{h}{4})^2 h
\]

\[
V = \frac{\pi}{16} h^3 \quad \frac{dV}{dt} = \frac{3\pi}{16} h^2 \frac{dh}{dt}
\]

\[
\frac{dh}{dt} = \frac{32}{3\pi h^2}
\]

When \(h = 5\) cm:

\[
\frac{dh}{dt} = \frac{32}{3\pi (5)^2} \quad \text{cm/min}
\]

When \(h = 10\) cm:

\[
\frac{dh}{dt} = \frac{4}{3\pi (10)^2} \quad \text{cm/min}
\]

5. An air traffic controller spots two planes at the same altitude converging on a point as they fly at right angles to each other (see figure). One plane is 150 miles from the point moving at 450 miles per hour. The other plane is 200 miles from the point moving at 600 miles per hour.

(a) At what rate is the distance between the planes decreasing?

(b) How much time does the air traffic controller have to get one of the planes on a different flight path?

\[
\frac{dx}{dt} = -450 \text{ mi/hr}
\]

\[
\frac{dy}{dt} = -600 \text{ mi/hr}
\]

\[
\frac{ds}{dt} = \frac{-450x - 600y}{s}
\]

\[
x = 150 \quad \text{and} \quad y = 200 \implies s = 250
\]

\[
s^2 = 150^2 + 200^2
\]

\[
\frac{ds}{dt} = \frac{-450(150) - 600(200)}{650}
\]

\[
\frac{ds}{dt} = -\frac{3750}{650} \text{ mi/hr}
\]
6. A boat is being pulled into a dock by means of a winch 12 feet above the water. Suppose the boat is moving at a constant rate of 4 feet per second. Determine the speed at which the winch pulls in rope when there is a total of 13 feet of rope out. What happens to the speed at which the winch pulls in rope as the boat gets closer to the dock?

\[
\frac{dx}{dt} = -4 \text{ ft/sec}
\]

\[
r^2 = x^2 + 12^2
\]

\[
\frac{dr}{dt} = \frac{x}{r} \frac{dx}{dt}
\]

\[
2r \frac{dr}{dt} = 2x \frac{dx}{dt}
\]

\[
\frac{dr}{dt} = -\frac{4x}{r}
\]

\[
12^2 + x^2 = 13^2
\]

\[
x = 5
\]

\[
S = 12 \Rightarrow x = 5 \Rightarrow \frac{dr}{dt} = -\frac{4 \cdot 5}{12} = -\frac{5}{3}
\]

\[
\frac{dr}{dt} = -\frac{5}{3} \text{ ft/sec}
\]

* As the boat gets closer to the dock, \( x \to 0 \)

\[
\lim_{x \to 0} \frac{dr}{dt} = \lim_{x \to 0} \frac{-4x}{\sqrt{12^2 + x^2}} = 0
\]

\[
= 0 \text{ (as seen on graph)}
\]
7. An airplane flies at an altitude of 5 miles toward a point directly over an observer (see figure). The speed of the plane is 600 miles per hour. Find the rate at which the angle of elevation \( \theta \) is changing when the angle is (a) \( \theta = 30^\circ \), (b) \( \theta = 60^\circ \).

\[
\tan \theta = \frac{5}{x}
\]

\[
\sec^2 \theta \frac{d\theta}{dt} = -\frac{5}{x^2} \frac{dx}{dt}
\]

\[
\frac{d\theta}{dt} = -\frac{5 \cos^2 \theta}{x^2} \frac{dx}{dt}
\]

\( \theta = 30^\circ \)

\[
x = \frac{5}{\tan 30^\circ} = \frac{5 \cos 30^\circ}{\sin 30^\circ} = \frac{5 \sqrt{3}/2}{1/2} = 5\sqrt{3}
\]

\[
\frac{d\theta}{dt} = \frac{3000 \left( \frac{\sqrt{3}/2}{1/2} \right)^2}{5\sqrt{3}} = \frac{450}{\sqrt{3}} \text{ deg/hr}
\]

\( \theta = 60^\circ \)

\[
x = \frac{5}{\tan 60^\circ} = \frac{5 \left( \frac{\sqrt{3}/2}{\sqrt{3}/2} \right)}{5\sqrt{3}} = \frac{5}{\sqrt{3}}
\]

\[
\frac{d\theta}{dt} = \frac{3000 \left( \frac{1/2}{1/2} \right)^2}{5\sqrt{3}} = \frac{150}{\sqrt{3}} \text{ deg/hr}
\]
8. The cross-section of a 5-meter trough is an isosceles triangle with a 3-meter base, and an altitude of 3 meters. Water is running into the trough at a rate of 1 cubic meter per minute. How fast is the water-level rising when the water is 1 meter deep?

\[
\frac{dV}{dt} = 1 \text{ m}^3/\text{min} \\
V = \frac{1}{2} x^2 \cdot 5 \\
V = \frac{5}{2} x^2 \\
\frac{dV}{dt} = \frac{5}{2} x \frac{dx}{dt} \\
1 = \frac{5}{2} x \frac{dx}{dt} \quad \rightarrow \quad \frac{dx}{dt} = \frac{1}{5x} \quad \text{When } h = x = 1 \quad \frac{dx}{dt} = \frac{1}{5} \text{ m/min.}
\]